Sparse Two-Dimensional FIR Digital Filters Design Using FISTA

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***Abstract*—An efficient algorithm described in this brief is proposed for designing two-dimensional (2D) linear-phase finite impulse response (FIR) digital filters. This algorithm is inspired by the methods described in [11], [14]. The proposed algorithm handles the problem on $L_1 - L_2$ formula directly and does not recast the problem to Quadratic Programming (QP) shown in [11]. Moreover, the choice of trade-off parameter arises in $L_1 - L_2$ is studied in this paper. Two examples are presented to illustrate our new algorithm.**

***Index Terms*—$L_1 - L_2$, sparse, 2D, FIR, linear-phase.**

I. Introduction

In recent years, digital filters has been widely used in some engineering areas, e.g. communications and signal processing [1]. Since a large number of design problems can be cast to optimization problems[1], [2]. Among this, design of 2D filters plays an important role in multidimensional digital signal processing. Much progress has been made in the past four decades [3], [4]. FIR filters are better than infinite impulse response (IIR) filters in some areas, e.g. image processing, radar and sonar signal processing, because FIR filters are inherently stable and may have exact linear phase responses. Due to these merits, 2D FIR digital filters are studied in this paper.

In the traditional filter design algorithms, the efficient reliable numerical design methods are taken more attention, and the implementation efficiency is seldom taken into account during the design stage. In fact, digital filters with sparse coefficients have been investigated by several authors, such as specific system structures or specific classes of filters. Some were introduced by Lim et al. about frequency-response masking (FRM) filters [5] and extended to the 2D case [6]. In [7] Gaustafsson et al. studied the sparse half-band filter like FIR. More details can be found in [11], [14] and the references therein. Due to the development in compressive sensing [8]-[10], Lu and Hinamoto examine the sparsity in filter design stage [11], [14]. In their research the sparse design problem is formulated as a QP or Second-Order-Cone-Programming (SOCP) and then resolved by some existed solvers which will be introduced in Section IV. From our simulations this strategies have a slow convergence when the filter order is large. Moreover, works in [11], [14] involve a trade-off parameter and have not introduced how to choice it in their work.

The main contributions in this paper are as follows.

1) The design problem introduced in [11], [14] is cast to $L_1 - L_2$ problem. Moreover, we solve this problem directly and do not reformulate as QP described in [11]. A fast iterative shrinkage-thresholding algorithm (FISTA) is involved in solving the $L_1 - L_2$ problem. The FISTA is suitable for solving large-scale problem which leads to an efficient result, especially for high order filters. The simulation will demonstrate the effectiveness of this algorithm.
2) In order to solve the trade-off parameter, binary search is used in the design stage. From binary search, a well balance between the E_2 error and sparsity is presented. The definition of E_2 will be given in the following section.

All of the changes described above are demonstrated by the simulations proposed in Section IV.

This paper is organized as follows. The design criteria, some basic theory about 2D FIR filter and the sparse design stages are given in Section II. Section III contains our main contributions about the proposed algorithm and some useful comments. There are two examples presented in Section IV to illustrate our proposed algorithm. The examples show the feasibility and efficiency, respectively. Some conclusions are shown in Section V.

II. Preliminaries and Problem Formulation

Some preliminaries about linear-phase 2D FIR digital filters and sparse problem formulation are given in this section. Moreover, some criteria for the design problem are also proposed.

A. Criteria in designing digital filters

There are some criteria in designing digital filters. While in this paper the least square error between the desirable and actual filter's frequency response in the range of all interesting bands is taken into account. Without loss of generality, we take the one-dimension (1D) FIR digital filters designing in this criterion as an example. Assume $D(\bar{\omega})$ is the desirable filter

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and Ω_I represents the interested frequency bands. A dense grid of frequency points $\bar{\omega}_i (i = 1, 2, \cdots, K)$ are sampled evenly over Ω_I. The problem can be formulated as

$$\min_{\boldsymbol{h}} \quad E_2 = \frac{1}{K}\Big(\sum_{\bar{\omega}_i \in \Omega_I} |D(\bar{\omega}_i) - H(e^{j\bar{\omega}_i})|^2\Big)^{\frac{1}{2}}$$

where

$$H(e^{j\bar{\omega}}) = \boldsymbol{z}_{N-1}^{\mathcal{T}} \boldsymbol{h}$$

with the coefficient vector and fourier basic below

$$\begin{aligned} \boldsymbol{h} &= \begin{bmatrix} h_0 & h_1 & \cdots & h_{N-1} \end{bmatrix}^{\mathcal{T}} \\ \boldsymbol{z}_{N-1} &= \begin{bmatrix} 1 & z^{-1} & \cdots & z^{-(N-1)} \end{bmatrix}^{\mathcal{T}} \end{aligned}$$

for $z = e^{j\bar{\omega}}$.

This criterion is simply and efficiently used in the previous research. Furthermore, in this paper we design the sparse filter under this criterion constraint and find a good comprise in the sparse and E_2 error.

B. Transfer function and impulse response for linear-phase 2D digital filters

A linear-phase 2D FIR filter's transfer function can be represented as follows:

$$H(z_1, z_2) = \sum_{i=0}^{N_1-1} \sum_{j=0}^{N_2-1} H_{ij} z_1^{-i} z_2^{-j} = \boldsymbol{z}_1^{\mathcal{T}} \boldsymbol{H} \boldsymbol{z}_2 \tag{1}$$

where N_1 and N_2 are odd integers, H_{ij} represents the (i, j)th element of matrix $\boldsymbol{H}$, $\mathcal{T}$ denotes the transpose operation, and

$$\boldsymbol{z}_l = \begin{bmatrix} 1 & z_l^{-1} & \cdots & z_l^{-N_l+1} \end{bmatrix}^{\mathcal{T}}$$

with $l = 1, 2$. Due to the phase-response linearity described in (1), the coefficient matrix $\boldsymbol{H}$ can be denoted as:

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{h}_{12} & \boldsymbol{H}_{13} \\ \boldsymbol{h}_{21}^{\mathcal{T}} & h_{22} & \boldsymbol{h}_{23}^{\mathcal{T}} \\ \boldsymbol{H}_{31} & \boldsymbol{h}_{32} & \boldsymbol{H}_{33} \end{bmatrix} \tag{2}$$

where the submatrices and vectors depicted in (2) have some interesting symmetric relations. Moreover, the related relations are given below[14]:

$$\begin{aligned} \boldsymbol{H}_{11} &= \text{flipud}(\text{fliplr}(\boldsymbol{H}_{33})) \\ \boldsymbol{H}_{13} &= \text{flipud}(\boldsymbol{H}_{33}),\ \boldsymbol{H}_{31} = \text{fliplr}(\boldsymbol{H}_{33}) \\ \boldsymbol{h}_{12} &= \text{flipud}(\boldsymbol{h}_{32}) \quad \text{and} \quad \boldsymbol{h}_{21}^{\mathcal{T}} = \text{fliplr}(\boldsymbol{h}_{23}^{\mathcal{T}}) \end{aligned} \tag{3}$$

where flipud and fliplr mean the operation of flipping a matrix or vector upside down and from left to right, respectively. The dimension of the submatrices and vectors can be determined in context. From equation (1)(2)(3), the frequency response of the linear-phase 2D FIR filters can be cast to

$$H(\omega_1, \omega_2) = e^{-j(n_1\omega_1 + n_2\omega_2)} \boldsymbol{c}_1^{\mathcal{T}}(\omega_1) \hat{H} \boldsymbol{c}_2(\omega_2) \tag{4}$$

where $\boldsymbol{c}_l(\omega_l) = \begin{bmatrix} 1 & \cos\omega_l & \cdots & \cos n_l\omega_l \end{bmatrix}^{\mathcal{T}}$ with $l = 1, 2$, $n_1 = \frac{N_1-1}{2}$, $n_2 = \frac{N_2-1}{2}$ and

$$\hat{H} = \begin{bmatrix} h_{22} & 2\boldsymbol{h}_{23}^{\mathcal{T}} \\ 2\boldsymbol{h}_{32} & 4\boldsymbol{H}_{33} \end{bmatrix}$$

Due to the linear-phase, in the design procedure, we can only take the magnitude frequency response into account. Then, the magnitude frequency response of (4) is

$$A(\omega_1, \omega_2) = \boldsymbol{c}_1^{\mathcal{T}}(\omega_1) \hat{H} \boldsymbol{c}_2(\omega_2) \tag{5}$$

As analog to the design procedure in 1D digital filters, equation (5) can be equivalently transformed to

$$A(\omega_1, \omega_2) = \boldsymbol{c}^{\mathcal{T}}(\omega_1, \omega_2) \boldsymbol{h} \tag{6}$$

where $\boldsymbol{c}^{\mathcal{T}}(\omega_1, \omega_2)$ is a vector of length $(n_1 + 1)(n_2 + 1)$ determined the rows of $\boldsymbol{c}_2(\omega_2)\boldsymbol{c}_1^{\mathcal{T}}(\omega_1)$, and $\boldsymbol{h}$ is a vector generated by the columns of $\hat{H}$.

From these analyses, the all design procedures in 2D are very similar in the 1D. It is noted that this compromise will lead the problem very large when the order is high. However, this is not the core in this paper, we will not discuss it here.

C. Problem formulation

The number of non-zero coefficients, denote Ξ, in $\boldsymbol{h}$ is used to measure the sparsity of the filter. Generally speaking, we hope the value of Ξ as small as possible when $\boldsymbol{h}$ satisfies the required design under E_2. We denote a function $\|\boldsymbol{h}\|_0$ which means the value of Ξ in $\boldsymbol{h}$. The function $\|\boldsymbol{h}\|_0$ means the L_0-norm for convenience despite not being a true norm. Moreover, the true norm is given below and will be used in the sequel.

$$\|\boldsymbol{h}\|_p = \Big(\sum_{n=0}^{N-1} |h_n|^p\Big)^{\frac{1}{p}}$$

where, for $p \geq 1$, is known as the L_p norm and N is the number of entries in vector $\boldsymbol{h}$. Assume the desirable 2D FIR digital filter's zero-frequency response is $A_D(\omega_1, \omega_2)$. Thus, the sparse filter design can be transformed to

$$\begin{aligned} \min_{\boldsymbol{h}} \quad & \|\boldsymbol{h}\|_0 \\ \text{s.t.} \quad & \Big(\sum_{\omega_1}\sum_{\omega_2} |A_D(\omega_1, \omega_2) - A(\omega_1, \omega_2)|^2\Big)^{\frac{1}{2}} \leq \delta \\ & (\omega_1, \omega_2) \in (\Omega_{I1}, \Omega_{I2}) \end{aligned} \tag{7}$$

where Ω_{I1} and Ω_{I2} have the same mean with Ω_I.

It's well known that the problem described in equation (7) is an NP-Hard problem. In order to get an proper solution, the approximation is used. There are some useful approximation criteria proposed in [11], [12]. It is mentioned that the L_1 is an efficient and simple approximation under some special conditions [13] and will be used in this paper. Therefore, the problem can be transformed to

$$\begin{aligned} \min_{\boldsymbol{h}} \quad & \|\boldsymbol{h}\|_1 \\ \text{s.t.} \quad & \Big(\sum_{\omega_1}\sum_{\omega_2} |A_D(\omega_1, \omega_2) - A(\omega_1, \omega_2)|^2\Big)^{\frac{1}{2}} \leq \delta \\ & (\omega_1, \omega_2) \in (\Omega_{I1}, \Omega_{I2}) \end{aligned} \tag{8}$$

This problem can be solved by QP in [11] or SOCP in [14]. However, when the parameters are very large, e.g. more than 10^3, the former methods are very inefficient. Fortunately, this problem can be equivalently transformed to an $L_1 - L_2$ problem under a proper trade-off parameter γ, like

$$\min_{\boldsymbol{h}} \quad \tfrac{1}{2}\|A_D(\omega_1, \omega_2) - A(\omega_1, \omega_2)\|_2^2 + \gamma\|\boldsymbol{h}\|_1 \tag{9}$$

Moreover, the $L_1 - L_2$ problem, even the parameters are large, can be efficiently solved by a family named iterative-shrinkage algorithms which shown in [15]. In this paper the FISTA algorithm is involved in solving our proposed problem (9). More details will be given in the following section.

Traditionally, the design of a sparse filter is done in two steps. The first step is at determining the locations of the coefficients which should be set to zero to reach the sparsity requirement. It is mentioned that the solution from (9) is a sparse solution and then hard-thresholding with an proper ε is used to $\boldsymbol{h}$ to yield an $\hat{\boldsymbol{h}}$ with the desired sparsity. Moreover, we denote the locations of zero-valued coefficients in the set S_{I^*}. In phase 2 of the design, we re-optimize the design problem which constrains by $h_n = 0,\ n \in S_{I^*}$ under E_2 error. The problem is given below for convenience.

$$\begin{aligned} \min_{\boldsymbol{h}} \quad & \|A_D(\omega_1,\omega_2) - A(\omega_1,\omega_2)\|_2 \\ \text{s.t.} \quad & h_n = 0,\ h_n \in S_{I^*} \end{aligned} \tag{10}$$

III. The Proposed Algorithm

According to the former discussions, the problem has been formulated. In this section we will give out our proposed algorithm which can solve the problem efficiently and determine the proper trade-off parameter γ which has not been shown in [11][14].

With a determined γ, the problem in (9) can be solved by FISTA which describes in [16], [17]. The simulations demonstrate that this algorithm is more efficient than the methods described in [11], [14]. For convenience, the FISTA algorithm is given below,

FISTA:

Input: Initial point $\boldsymbol{h}_0$, trade-off parameter γ, the desirable zero-phase frequency $A_D(\omega_1,\omega_2)$, Lipschitz constant $L = \lambda_{\max}(\boldsymbol{C}^{\mathcal{T}}\boldsymbol{C})$ and number of iterations M.

Step 1 Set $\boldsymbol{z}_1 = \boldsymbol{h}_0$, $t_1 = 1$, $m = 1$.

Step 2 Compute $\boldsymbol{h}_m = S_{\frac{\gamma}{L}}\{\frac{1}{L}\boldsymbol{C}^{\mathcal{T}}(A_D(\omega_1,\omega_2) - \boldsymbol{C}\boldsymbol{z}_m) + \boldsymbol{z}_m\}$.

Step 3 Compute $t_{m+1} = \frac{1+\sqrt{1+4t_m^2}}{2}$.

Step 4 Update $\boldsymbol{z}_{m+1} = \boldsymbol{h}_m + \frac{t_m - 1}{t_{m+1}}(\boldsymbol{h}_m - \boldsymbol{h}_{m-1})$.

Step 5 If $m < M$, set $m = m+1$ and repeat from *Step 2*; otherwise stop and output $\boldsymbol{h}_k$ as the solution.

Comments:

- The initial point described above always set to $\mathbf{0}$. The trade-off parameter γ can balance the value between $\|\cdot\|_2$ and $\|\cdot\|_1$. How to determine the proper γ will be proposed in the following parts. Moreover, the algorithm can be terminated by the number of iterations M or $\|\boldsymbol{h}_{k+1} - \boldsymbol{h}_k\|_2 \le \iota$. ι is the tolerance value.
- $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a matrix. The matrix $\boldsymbol{C}$ is composed with vector $\boldsymbol{c}^{\mathcal{T}}(\omega_1,\omega_2)$ which sampled on the interesting bands both ω_1 and ω_2. The operation $S_\alpha(\boldsymbol{u})$ means

$$S_\alpha(\boldsymbol{u}) = \begin{cases} \operatorname{sgn}(\boldsymbol{u})\max(|\boldsymbol{u}| - \alpha, 0), & |\boldsymbol{u}| \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

  where sgn, max denote the signa and maximum operation, respectively.
- From our experience, this algorithm used to solve the problem (9) is faster than the methods described in [11], [14].

How to determine the proper trade-off parameter γ is an important problem and not solved in [11], [14]. Moreover, in this paper binary search is used to solve this problem and the simulations demonstrate that this strategy is work. Without loss of generality, assume the design requirement is for a determined filter order $N-1$, the desirable zero-phase frequency response $A_D(\omega_1,\omega_2)$, hard-thresholding parameter ε, the value of zero-valued coefficients $\xi(\xi = N - \Xi)$, the trade-off parameter upper and low boundary γ_u, γ_l. This strategy is named as **TFS** in this paper.

TFS:

Input: The parameter N, $A_D(\omega_1,\omega_2)$, ε, ξ, γ_u, γ_l. Let $\gamma = \frac{\gamma_u+\gamma_l}{2}$.

Step 1 Running the algorithm FISTA described above, we denote the number of zero-valued coefficients by Δ with hard-thresholding parameter ε.

Step 2 Update the trade-off parameter by the following equation.

$$\begin{cases} \gamma_l = \gamma, & \text{if} \quad \Delta < \xi \\ \gamma_u = \gamma, & \text{if} \quad \Delta > \xi \end{cases}$$

Then, set $\gamma = \frac{\gamma_u+\gamma_l}{2}$ and repeat from *Step 1*. If $\Delta = \xi$, set the locations of zero-valued coefficients to S_{I^*} and stop the algorithm, output the final result as the solution.

Comments:

- Due to the unknown parameter γ, this strategy is efficiently used to solve the design problem. In order to find a proper γ, the FISTA algorithm may be run several times. However, the algorithm is also efficiently and demonstrated by the simulation.
- The problem described in (9) is a convex problem. Whatever the initialization is, the global solution will be find. However, a better initialization will speed up the convergence. Due to this fact, we set the solution of the former γ to the initialization of the next updated γ. This trick is very useful in the simulation.

After the former procedure, the first phase in designing sparse filter is completed. In the second phase, we can eliminate the equality constrains and then the problem is recast to a simple QP problem which has no constrains. This problem has an analytical solution through pseudo-inverse which can refer in [11] for more details.

IV. Design Examples

In this section, two examples are presented to demonstrate the effectiveness of the proposed algorithm. All designs described in this section are conducted on a desktop computer with an Intel Xeon(R) E5-2670 @2.6GHz.

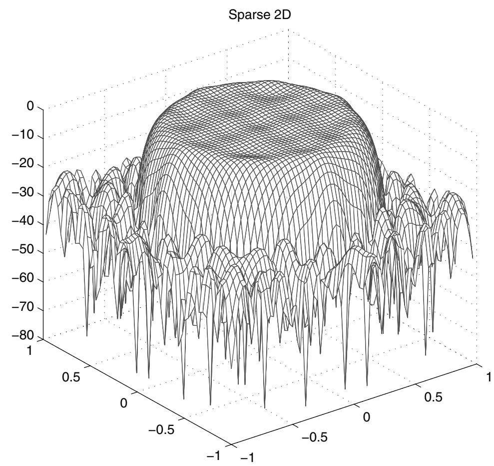

Fig. 1. Sparse 2D circularly symmetric lowpass filter of size 23×23 with 480 zero coefficients.

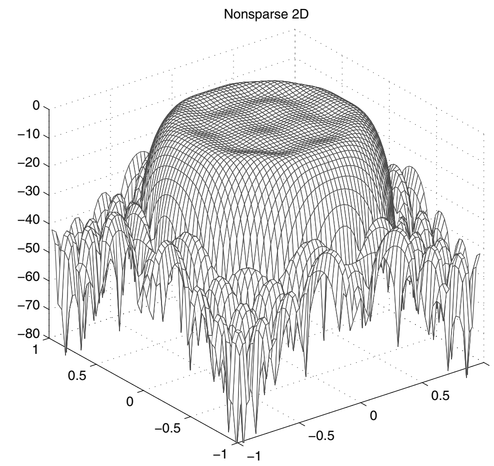

Fig. 2. Equivalent nonsparse filter of size 11×11 with nonzero coefficients.

A. Example I

In this example a circularly symmetric lowpass 2D linear-phase FIR sparse digital filter is given. The design specific is as follows, $N_1 = N_2 = 23$, $\varepsilon = 0.025$, $\gamma_u = 1$, $\gamma_l = 0$, $M = 20$, $K = 300$, $\xi = 408$,

$$A_D(\omega_1, \omega_2) = \begin{cases} 1, & \omega_1^2 + \omega_2^2 \leq 0.5 \\ 0.1, & 0.5 < \omega_1^2 + \omega_2^2 < 0.7 \\ 0, & \text{otherwise} \end{cases}$$

Run the **TFS**, the algorithm converges after 1 iteration. The E_2 error is 3.71×10^{-4}. The performance of the sparse filter is compared with the equivalent least square optimal nonsparse filters that contains the same number of nonzero coefficients as in their sparse counterparts. See the magnitude responses in figures 1 and 2. Moreover, the E_2 error in nonsparse filter is 3.89×10^{-4}. From the E_2 error criterion, we know the sparse filter outperform the nonsparse filter with the same number of nonzero coefficients.

B. Example II

This example is used to demonstrate the efficiency of FISTA used to solve the problem (9). Compared with the method proposed in [11], the FISTA has a more fast convergence. The designed parameters are same in IV-A and the filter order is $N_1 = N_2 = 5, 7, \cdots, 29$. The toolbox Sedumi [18] and quadprog in MATLAB are used to solve the problem described in [11]. When the L_2 norm among the solutions formulated by the three solvers less than 10^{-7}, we think the all solvers output the trust solution. It is noted that the all initialization is set to $\boldsymbol{h} = 0$. The cputime with the different orders are presented in figure 3.

It is mentioned that the cputime with quadprog for order $N_1 = N_2 = 29$ is not presented in the figure because in this order the quadprog can not get a trust solution. What's more, from the figure we know the FISTA is far more efficient to solve the problem in (9), especially when the order is high.

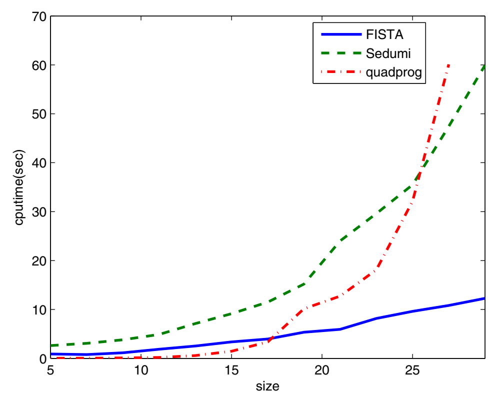

Fig. 3. The efficiency of the different algorithms.

V. Conclusion

A new algorithm is proposed in this paper. Due to the efficient converging speed, the algorithm is used for designing 2D sparse linear-phase FIR filters, even when the order is high. Moreover, a heuristic strategy is involved in handling the choice of trade-off parameter in $L_1 - L_2$ problem. The simulations demonstrate the efficiency and effectiveness of the proposed algorithm. From our experience, the algorithm also can be used for designing non-linear-phase filter. In the future, finding an algorithm which can search more zero-valued coefficients is an interesting topic. In our research, some new exciting results have been found, and we hope the new results will be published in the future. The related research is still ongoing.

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